Pragmatic TCM for 8-PSK in Satellite Communications

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Abstract

To improve satellite transponders' bandwidth utilization, trellis-coded modulations using an "industry standard" 64-state, rate 1/2 convolutional code on *M*-ary PSK constellations have be proposed and patented. We provide implementation details: simpler branch metric computations for a DSP-based 8-PSK trellis decoder and a phase ambiguity resolution method using the R-S outer code.

1 Introduction

Intelsat has adopted a new standard [1], which employs trellis coded modulation (TCM), using 8-PSK, with mandatory Reed-Solomon (R-S) (219, 201) outer coding over GF(2^8). This standard has twice the bandwidth efficiency (at almost 2 bps/ Hz) than the IESS-308 standard it replaces (at almost 1 bps/Hz). It states: "Since 8-PSK TCM uses practically the same satellite power[1] and is twice as bandwidth efficient, its usage will permit more efficient use of orbital spectrum". This standard, reviewed in section 2, does not use the optimum 64-state Ungerboeck TCM [2] for the chosen 8-PSK modulation. Rather, a "pragmatic" TCM (PTCM) scheme, based on the methodology of [3] and two patents [4,5] was chosen. The use of PTCM is justified in [3] as follows:

1) There exists a widely used "industry standard" constraint-length 7 (64-state), rate 1/2 convolutional code that is optimum for BPSK and QPSK.
2) While the use of this convolutional code in PTCM results in a 2dB clear sky[2] loss relative to the optimum 64-state, rate 2/3 Ungerboeck TCM at extremely low BERs, there is only a 0.4dB loss at a BER of 10^{-5}. The mandatory R-S outer code further reduces this BER to an acceptable level.

One patent [4] title reflects the chief benefit of PTCM for 8-PSK: reduction of the traceback memory and computation complexity[3] (per decoded bit) associated with the PTCM decoder relative to a decoder for the optimum 64-state Ungerboeck TCM decoder. It also describes a metric setting method, reviewed in section 3, that requires a conversion from in-phase and quadrature data to phase (this requires a divide, a table look-up and other four-quadrant logic to be provided external to the "industry-standard" Viterbi decoder). Section 4 describes a simple metric setting procedure, suitable for DSP software implementation, yielding the desired performance using only multiplies and saturation logic.

The second patent [5] describes the phase ambiguity resolution circuit required if the PTCM scheme is used by itself (i.e., without Reed-Solomon outer coding). The use of this circuit effects the branch metric computation at low to moderate E_b/N_0. The multiplication of errors caused by the ambiguity resolution circuit may be minimized by erasing (setting to 0) some branch metrics when the received signal is close to intermediate significant bit (ISB) transitions.

[1] At channel capacity, a bandwidth constrained channel with data rate to bandwidth ratio, *r*, requires a minimum $E_b/N_0=(2^r-1)/r$; thus, minimum E_b/N_0s are 0dB and 1.76dB at 1 bps/Hz and 2bps/Hz respectively. While there is no E_b/N_0 difference between uncoded BPSK (1bps/Hz) and uncoded QPSK (2bps/Hz) at any BER, the E_b/N_0 difference between rate 1/2 coded QPSK (1bps/Hz) and rate 2/3 PTCM using 8-PSK (2 bps/Hz) at a BER of 10^{-5} is -2dB.

[2] For clear sky, the satellite channel exhibits negligible ISI; rain may induce some fading..

[3] This TCM decoder's computational complexity per decoded bit is almost half as that of the 64-state rate 1/2 coded QPSK.

In section 5, two proposals for PTCM schemes at 2.5 bps/Hz[4] using 8-PSK are reviewed. The performance of simple branch metric computations for the more promising scheme is provided. In section 6, a procedure that uses the R-S outer code to resolve phase ambiguity is described.

2 Background

Figure 1 shows the PTCM phase ambiguity resolving encoder proposed in [4,5].

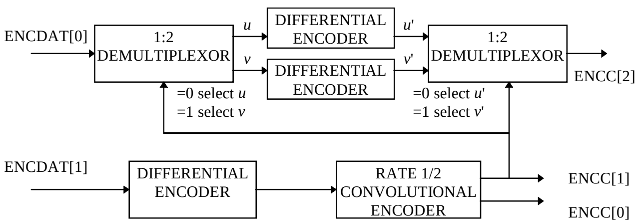

Figure 1. Phase Ambiguity Encoder for rate 2/3 PTCM using 8-PSK

The 8-ary symbol (bits ENCC[2:0]) is mapped to the 8-PSK constellation as shown in Figure 2.

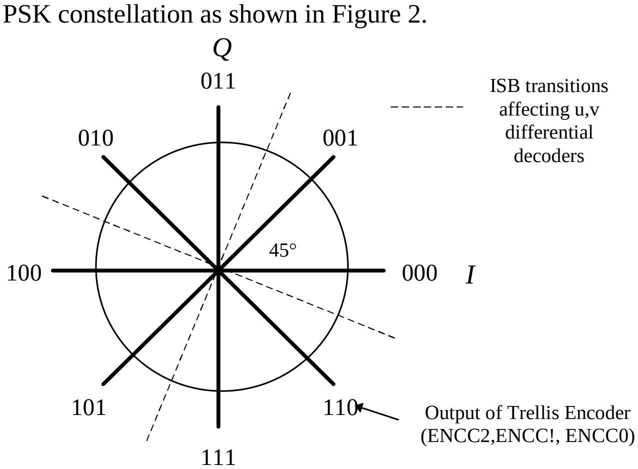

Figure 2. Symbol mapping to 8-PSK constellation

A block diagram of the PTCM decoder is depicted in Figure 3. We are principally concerned with a description and simplification of the first module of Figure 3: the computation of the branch metrics to the Viterbi Decoder.

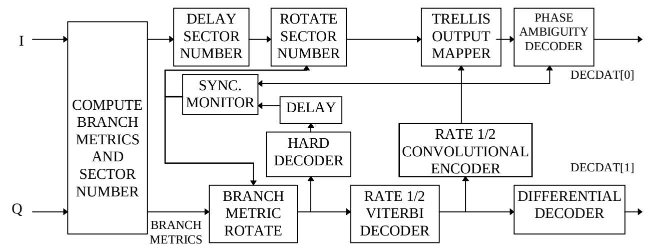

Figure 3. PTCM Decoder block diagram

We first consider the BER without the effect of error multiplication by the phase ambiguity resolution decoder. It is usual to compare the performance of a rate $(N\text{-}1)/N$ PTCM scheme using $M=2^N$ signals with the performance of an equivalent bandwidth uncoded system using $M/2$ signals. For uncoded operation, the BER, P_{bu} is bounded by[5] [3]:

$$0.5 \cdot erfc\left[\sqrt{\frac{E_s}{N_0}\sin^2\left(\frac{2\pi}{M}\right)}\right] < P_{bu} < erfc\left[\sqrt{\frac{E_s}{N_0}\sin^2\left(\frac{2\pi}{M}\right)}\right] \quad (1)$$

while the coded BER, P_{bc}, with an M-signal constellation is lower bounded by:

$$P_{bc} > \frac{K}{2} \cdot erfc\left[\sqrt{\frac{E_s}{N_0}\sin^2\left(\frac{4\pi}{M}\right)}\right] \quad (2)$$

where $K \leq 1$. For rate 2/3 PTCM using 8-PSK, using a standard 64-state convolutional code, this reduces to:

$$P_{bc} > 0.25 \cdot erfc\left[\sqrt{\frac{2E_b}{N_0}}\right] \quad (3)$$

(3) may be understood as follows: the minimum distance path is only one branch long, because of the two parallel transitions from a state, X, at stage n to stage n+1 (Figure 4). The single branch error probability is merely the BPSK bit error probability, with energy doubled since two bits are sent per symbol, and multiplied by a factor of 0.5 because only one out of two input bits is involved in such single branch decision errors. This is a lower bound because errors from multi-branch paths must also be considered. At higher E_b/N_0, for the standard 64-state code, the multi-branch errors may be neglected (in comparison to single branch errors) [3].

[4] [3] describes a PTCM at 3 bps/Hz using 16-PSK; schemes using higher-order modulations must be carefully evaluated with respect to their sensitivity to phase error and spectral regrowth (when non-linearly amplified). 2.67 bps/Hz PTCMs using 8-PSK (with 1 dB E_b/N_0 penalty when compared to PTCMs at 2.5 bps/Hz) using punctured codes (derived from the standard rate 1/2 code) are also described by [7] and [8].

[5] $erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{u^2} du$

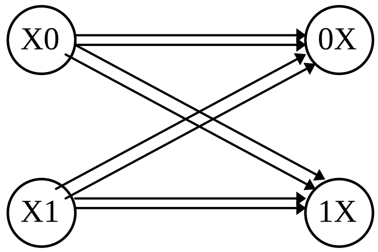

Figure 4. Trellis for 8-PSK pragmatic TCM

To achieve this performance, the branch metrics must be set according to the Euclidean distance between received signal with respect to the 4 closest transmitted points in the signal constellation.

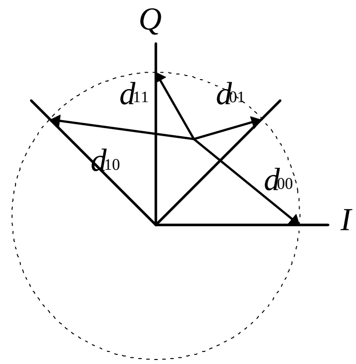

Figure 5. Branch metrics calculated using Euclidean distances to four nearest neighbors.

3 Branch metric Computation

For gray-coded QPSK used in IESS-308, with the constellation points being $e^{jk\pi/2}$, k=0,1,3 and 2, four (soft decision) branch metrics computed from the incoming in-phase and quadrature matched filter outputs, *I* and *Q*, are simply *I*, *Q*, *-I* and *-Q* corresponding to symbols 00, 01, 11 and 10. These matched filter values, when negated, may be thought of as relative squared Euclidean distances[6].

For IESS-310, without loss of generality, we consider received matched filter pair as shown in Figure 5 and the squared Euclidean distances between received signal with respect to the 4 closest transmitted points in the 8-PSK constellation. The squared distances with respect to constellation points on a radius *R* circle are:

$$\begin{aligned}
d_{00}^2 &= (I-R)^2 + Q^2 = P - 2IR \\
d_{01}^2 &= (I-\tfrac{R}{\sqrt{2}})^2 + (Q-\tfrac{R}{\sqrt{2}})^2 = P - \sqrt{2}(I+Q)R \\
d_{11}^2 &= I^2 + (Q-R)^2 = P - 2QR \\
d_{10}^2 &= (I+\tfrac{R}{\sqrt{2}})^2 + (Q-\tfrac{R}{\sqrt{2}})^2 = P - \sqrt{2}(Q-I)R
\end{aligned} \quad (4)$$

R (which, assuming no fading, is constant) may be estimated by an automatic gain control circuit (AGC).

[6] These values may be negative, and therefore, are not true "distances"; however, only relative distances matter to the Viterbi decoder.

If the effects of varying *P* ($=I^2+Q^2+R^2$) are ignored, the branch metrics may be taken to be the last terms in the right hand side of (4), the correlations of the received signal with the closest 4 transmitted signals. In order to avoid determining which 4 signals (of the 8) are closest to the received signal, absolute values of the correlations of the received signal with only those vectors depicted in Figure 4 may be used. The 4 correlations may be computed using 2 adds, 2 multiplies by a constant, 4 comparisons, and, in the worst case, 4 negations.

There are two difficulties with directly using the correlations computed above in a Viterbi decoder:

a) When $P \gg 2R^2$ or $P \ll 2R^2$ (due to noise), so are the correlations (the relative distances), and these are given undue weight in a finite horizon Viterbi decoder
b) ISB decoding (ISB hard decision boundaries are shown in dashed lines in Figure 2) errors cause error multiplication in the phase ambiguity resolution circuit[7].

To resolve the first difficulty, the correlations have upper and lower limits applied to them. To resolve the second difficulty, erasures (0s) take the place of correlations with respect to the farthest 2 (of the 4 closest) transmit signals when the received signal is close to these ISB decision boundaries.

[7] There are two sources of error multiplication: the first due to binary differential decoding and the second due to incorrect demultiplexor selection caused by ISB errors. The explanation of (3) provided in section 2 shows that error multiplication due to ISB errors is of significance only at low to moderate E_b/N_0.

In current practice, the I and Q samples are first converted to an angle (using a division and a 4-quadrant arctangent table look-up) and then the 4 metrics are set according to a table (for example, this procedure is followed in Qualcomm's PTCM decoder). Note that these metrics have two periods in $(0,2\pi]$ (due to taking absolute values of correlations).

4 Efficient Branch metric Computation

Motivated by the periodicity of the correlations described above, more efficient metric calculations are:

$$\begin{aligned} d_{00}^2 &= -(I^2 - Q^2) \\ d_{01}^2 &= -2IQ \\ d_{11}^2 &= (I^2 - Q^2) \\ d_{10}^2 &= 2IQ \end{aligned} \qquad (5)$$

These surrogate squared distances[8] may be computed using 3 multiplies[9]. These are then limited symmetrically with respect to 0 (this involves an additional 8 comparisons and, in the worst case, 4 substitutions). The limit value can be so chosen that the relative distances are, for all practical purposes, the same as the relative distances in (4) after limiting. For simplicity, modification of metrics for received signals close to ISB transitions are omitted[10].

The performance of this metric setting procedure, using 11-bit quantized[11] I and Q values, a traceback memory of 38 states and empirically optimized metric saturations, shown in Figure 6, approaches the theoretical lower bound at high E_b/N_0 and is comparable to a commercially available PTCM decoder. Details of the Viterbi decoder used for this implementation are provided in [6].

[8] Approximate distances are adequate at all E_b/N_0's where single branch errors dominate (where (3) applies).
[9] Each multiply is typically a single cycle operation on modern DSPs.
[10] This refinement may be introduced with a small additional complexity, but the gain at moderate to high SNRs is insignificant.
[11] 11-bit quantization is used for proper sector decoding; 6 bit quantized I and Q values are adequate for the Viterbi decoder.

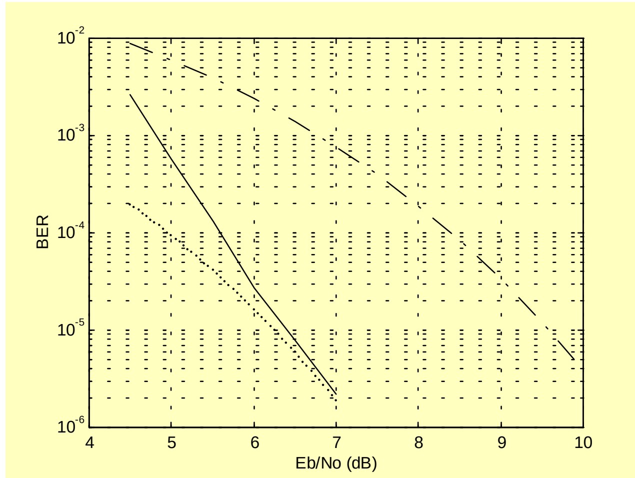

Figure 6. Rate 2/3, 8-PSK PTCM performance using the simplified branch metric computation (solid line) compared to uncoded QPSK (dashed line). The bound of (3) is the dotted line.

5 PTCM at 2.5 bps/Hz using 8-PSK

A rate 5/6 code for 8-PSK using a "industry-standard" rate 1/2, 64-state convolutional encoder punctured to rate 3/4 is described in [7] as shown in Figure 7. The normalized square Euclidean distance for this code is 1.465 (the punctured code has a free Hamming distance of 5). Thus, though this PTCM provides 2.5 bps/Hz as compared to 2 bps/Hz for uncoded QPSK, it still provides an asymptotic coding gain (ACG) of 1.66dB.

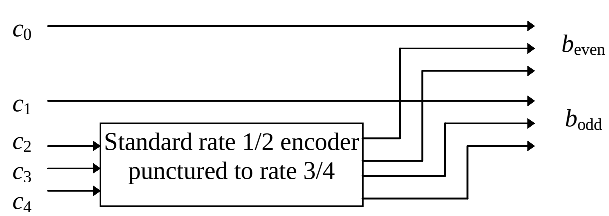

Figure 7. Rate 5/6 PTCM using a punctured "industry-standard" rate 1/2 code. The odd and even sets of tribits are mapped as in Figure 2.

A phase ambiguity resolution circuit based on the similar ideas as shown in Figure 2 may be used. However, in [8] it is pointed out that, when mandatory R-S outer coding is used, it may be used to resolve phase ambiguity. This avoids error multiplication due the phase ambiguity resolution circuit. The PTCM scheme used in [8] does not use a punctured 64-state code; instead, the pair of bits produced by the convolutional code are time interleaved on odd and even bauds as shown in Figure 8.

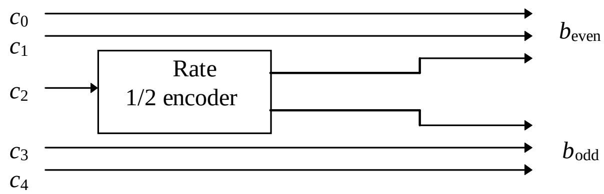

Figure 8. Rate 5/6 PTCM using an unpunctured rate 1/2 code. The odd and even sets of tribits are mapped to the constellation in lexicographic order (rather than the gray coded order of Figure 2).

For this rate 5/6 PTCM using 8-PSK, using a 64-state convolutional code, the BER at high SNRs is:

$$P_{bc} > 0.4 \cdot erfc\left[\sqrt{\frac{1.25E_b}{N_0}}\right] \qquad (6)$$

(6) may be understood as follows: the minimum distance path is only one branch long, because of the two parallel transitions from a state, X, at stage n to stage n+1 (Figure 4). The single branch error probability is merely the QPSK bit error probability, with energy multiplied by 1.25 since 2.5 bits are sent per symbol as compared to 2 bits in QPSK, and multiplied by a factor of 0.8 because four out of five input bits are involved in such single branch decision errors. At higher E_b/N_0, for the 64-state code, the multi-branch errors may be neglected (in comparison to single branch errors). Thus, the ACG is $10\log_{10}(1.25)$=0.97dB which is worse than TCM obtained using a rate 1/2 code punctured to rate 3/4 by 0.69dB.

However, [8] states that ACG is not the sole criterion used in selecting a TCM scheme; rather the coding gain at the operating range of BERs should be considered[12]. As the BER of the scheme in [7] is ultimately limited by the rate 3/4 punctured code[13], it exhibits a sharper "knee" than the scheme of [8]. Thus, in a range of BERs (typically between 10^{-3} and 10^{-5} where R-S outer coding further reduces BERs to make them acceptable) [8]'s scheme performs better than [7]. The performance of the scheme of [8] is shown in Figure 9.

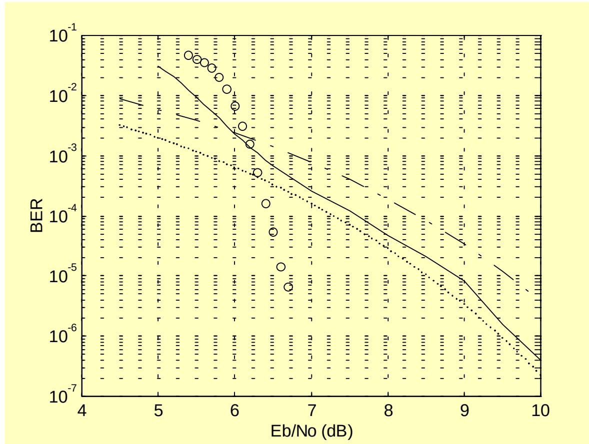

Figure 9. Rate 5/6, 8-PSK PTCM performance (solid line) compared to uncoded QPSK (dashed line). (6) is shown as a dotted line. The circles show the performance with a (225,205) Reed-Solomon outer code over GF(2^8).

6 Phase ambiguity resolution using R-S outer code

As suggested in [8], phase ambiguity (as well as symbol pair sychronization in the PTCM of Figure 8) may be resolved using the R-S outer codes with periodically inserted unique words (UWs) (avoiding error multiplication in phase ambiguity resolution circuits). For example, [1] prescribed mandatory R-S (219, 201) outer coding over GF(2^8) with periodically inserted unique words, but curiously includes a testing requirement as follows:

> "Due to the steepness of the BER versus E_b/N_0 response curve when using the Reed-Solomon outer coding, an inordinately long period of time is necessary to detect a sufficient number of errors to determine the BER performance with a reasonable degree of confidence at even moderate E_b/N_0 values. Assuming that Reed-Solomon outer codec is functioning, determining the BER performance of the TCM codec without Reed-Solomon outer coding, would enable users to quickly determine whether or not the modem is functioning correctly".

Evidently, any scheme that uses the R-S synchronization pattern to resolve phase ambiguities cannot cater to testing without R-S outer coding. Furthermore, the scheme of [8] avoids the multiplication of errors caused by the ambiguity

[12] This is essentially the same argument made in [3] in comparing its rate 2/3 PTCM scheme for 8-PSK with that used by the optimum Ungerboeck code.

[13] Since the rate 3/4 code's decoder requires twice the traceback memory of the unpunctured rate 1/2 decoder, [8] has this additional, but unclaimed, advantage (that [4]'s title gives importance to) over [7]. The results shown in Figure 9 were obtained using a Viterbi decoder with a traceback memory of 38 states.

resolution circuit described in [5]. As seen in section 4, setting of branch metrics in the scheme of [3] due to an ambiguity resolution circuit is also made more complex. However, the absence of a phase ambiguity resolution circuit may allow the inner code, for some repetitive data patterns, to indicate node synchronization, but the outer code to fail to synchronize[14]. The following procedure (assume that inner and concatenated codes are not synchronized and *timer*=0 initially) ensures synchronization with all data patterns:

```
if (inner code synchronized)
    if (outer code errors in s-bit UW< r)
        concatenated code is synchronized
    else {
        set inner code is not synchronized;
        increment inner code phase reference by 2π/M (mod 2π);
        if (phase==0) change symbol pair alignment;
    }
else
    if (timer++==timeout) {
        timer=0;
        increment inner code phase reference by 2π/M (mod 2π)
        if (phase==0) change symbol pair alignment;
    }
```

The inner code usually correlates the re-encoded decoded sequence and the (suitably delayed) hard-decision decoded received symbols in order to determine phase synchronization. The expected outer code synchronization time using this method, calculated using the methods described in [9], is not significantly different than the outer code synchronization time when the inner code incorporates a phase ambiguity resolution circuit such as [5].

7 Conclusion

The performance of a simplified metric setting procedure for 2 and 2.5 bps/ Hz PTCM decoder, suitable for DSP implementation,[15] is described and shown to be comparable to that provided by a commercially available PTCM decoder chip. We also extend this procedure to one 2.5 bps/Hz PTCM proposal. A method for phase ambiguity resolution and/or symbol set alignment using the R-S outer code unique words, that is data pattern insensitive, is also described. In summary, the trade-off criteria used in selecting a TCM scheme are:

- E_b/N_0 at operating BER versus decoder memory/ complexity
- Performance loss associated with phase ambiguity resolution methods versus synchronization time
- bps/Hz versus sensitivity to phase error/ spectral regrowth

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[9] S. Jayasimha and T. Praveen Kumar, "Synchronizing Concatenated FECs," Proc. of *National Conference on Communications (NCC-2000)*, pp. 13-16.

[14] For example, with an all zeroes pattern interrupted occasionally by unique words (for R-S synchronization), the inner code may declare node synchronization and yet produce an (incorrect) output pattern that prevents outer code synchronization.

[15] A TMS320C5402 implementation of the scheme of [3], including phase ambiguity resolution, can support data rates exceeding 1Mbps, while consuming 780 16-bit words of data memory and 1533 words of 16-bit program memory.