# An Adaptive IIR Phase Equalizer for Electronic Energy Meters

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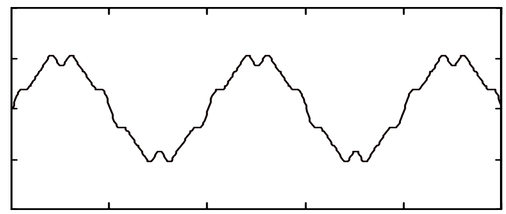
*Abstract*: International Standards for static watt-hour meters require wideband phase equalization of current/ voltage transformer (CT/ VT). The proposed adaptive filter structure equalizes CT/ VT (whose responses are near-ideal, but whose transfer function parameters are unknown) phase without altering their magnitude responses. A calibration procedure, together with its performance measures, is presented.

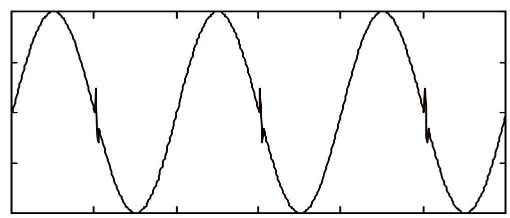
## 1. Introduction

As electronic meters are superior to their electromechanical predecessors in terms of accuracy, billing, tamper-detection capabilities, flexibility, manufacturability, reliability and cost, their use in power system networks is becoming widespread. The International Electrotechnical Commission (IEC) recommends two standards for static watt-hour meters (based on maximum percentage of error). IEC-687 [1] applies to 0.2% and 0.5% class meters while IEC-1036 [2] applies to 2% and 1% accuracy for watt-hour measurement. The American National Standards Institute's ANSI C12.16-1991 [3] specifies 'S' and 'A' classes of accuracy for watt-hour measurements. Accuracy requirements on reactive and apparent energies are generally specified by national standards [4]. Draft standards also require the meter to register line frequency energy and total energy (line frequency energy plus harmonic energy) separately.

Line voltage distortion (pollution) occurs when users draw large harmonic currents. Current measurement should be accurate when up to 30% total harmonic distortion is present [4]. Harmonics required for conformance testing to BS5406 and EN60555-2[5] (standards for disturbances in power supplies caused by household appliances, that specify current thresholds up to the 31$^{st}$ harmonic) should also be measured.



a. Harmonic Distortion



b. Notching

Figure 1. Quasi-stationary distortion in supply voltage

Electronic energy meters are also expected to have field programmable current transformer (CT) and voltage transformer (VT) turns ratios and input-output maps. Thus, when a meter is connected to external CTs and VTs, it can equalize VT/ CT phase and CT non-linearity [4]. Residual phase error (after phase equalization) between CT and PT paths, at any power factor and multiples of line frequency, should not exceed 1°. An adaptive procedure that equalizes CT and VT phase over a range of frequencies is described.

## 2. Adaptive IIR structure for phase equalization

Lumped linear component models for CTs and PTs are shown in Figure 2 (after translating secondary circuits to the primary), where $R_p$=primary resistance, $L_p$=primary inductance, $R'_s$=primary equivalent of secondary resistance, $L'_s$=primary equivalent of secondary inductance, $R'_e$=primary equivalent of load resistance, $I_o$=no load current, $E_p$ =primary induced voltage and $E'_s$ =primary equivalent of secondary induced voltage. In terms of these parameters, the transfer functions are:

$$P(s) = \frac{V_s}{V_p} = (n_p s L_o R_e^{'}) / \{s^2[L_s^{'}(L_o + L_p) + L_o L_p] + s[L_s^{'} R_p + (R_e^{'} + R_s^{'})(L_o + L_p) + L_o R_p] + R_p(R_e^{'} + R_s^{'})\} \quad (1)$$

where, $n_p$<1.

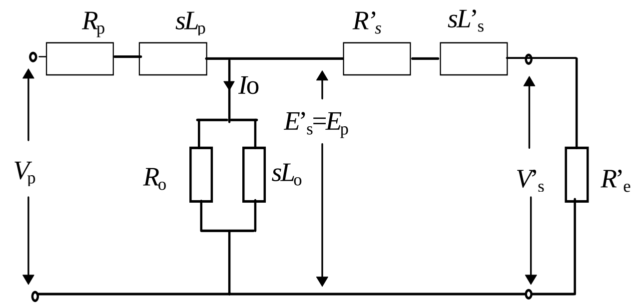
$$C(s) = \frac{V_s}{I_p} = \frac{n_c s L_o R_e^{'}}{s(L_o + L_s^{'}) + (R_s^{'} + R_e^{'})}, \; n_c > 1 \quad (2).$$

where $n_p$ and $n_c$ are the secondary to primary turns ratios of PT and CT. Due to manufacturing tolerances, the parameters R and L are known only approximately, and in many cases, all that is known is that the CT/ PT transfer functions are close to unity. The PT and CT transfer functions are suitably discretized (the bilinear transform or the impulse invariance methods [6]). Using the bilinear transform, the discrete time transfer functions are:

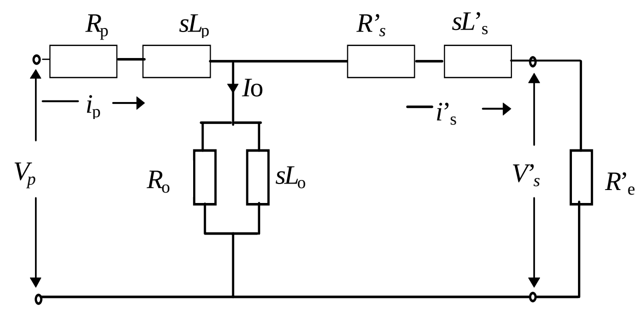
$$H_c(z) = \frac{b_{0c} + b_{1c} z^{-1}}{1 + a_{0c} z^{-1}} \quad (3)$$

$$H_p(z) = \frac{b_{op} + b_{1p} z^{-1} + b_{2p} z^{-2}}{1 + a_{0p} z^{-1} + a_{1p} z^{-2}} \quad (4)$$

where estimates of $a_c$'*s* and $b_c$'*s* are refined by the adaptive system of Figure 3. If $H_p(z)$ follows C(*s*) and $H_c(z)$ follows P(*s*), phase is equalized without significant amplitude distortion (as C(*s*) and P(*s*) are all-pass) in the desired frequency-range.




(a) PT Model




(b) CT Model

Figure 2. PT and CT Model

The model parameters are estimated during the calibration process by applying a wideband excitation to both the CT and VT after initializing the digital IIR CT and VT filter parameters (of Figure 3) to nominal values. In many cases of interest, the PT transfer function can be assumed to be known. This is because a voltage transformer's input voltage varies only within ±20% of a nominal voltage (the line voltage), while a current transformer should be accurate from 0.5% to 120% of the rated value. Thus, for a CT, a single model (using lumped linear components) is not accurate at all amplitudes[1].

A Hilbert transform (with a passband covering the range of frequencies of interest) pair follows the IIR filter pair, *P*(*z*) and *C*(*z*), the outputs of which are multiplied to obtain an error signal which adjusts the parameters of the IIR filter *C*(*z*) using a variation of the output error method [7].The latter method,

[1] When there is significant non-linearity in the CT, phase calibration at a few current set points may be required.

where the mean square error of the difference between $x_1$(n) and $x_2$(n) is minimized, could be used if one of the transfer functions is close to ideal (a transfer function of unity). As described in the next section, this assumption can be relaxed if the variation on the output error method described by Figure 3 is used. Also, the method of Figure 3 equalizes phase by only making minor perturbations to the initial parameter values, rather than the (in general) large perturbations needed for both magnitude and phase equalization of the output error method. The computational requirements of Figure 3 (executed during calibration only) is dominated by the Hilbert transformer, while the IIR filters (executed during normal operation as well) are of low complexity.

In Figure 3, if $P(z)$ is initialized to $H_p(z)$, and C(z) adapts to $H_c(z)$, $y_1(n)$ and $y_2(n)$, are orthogonal and the expected value of $e(n)$ is zero. However, if $C(z)$ is different from $H_c(z)$, the outputs $y_1(n)$ and $y_2(n)$ will not be orthogonal, and $e(n)$ can be used to adjust coefficients of $C(z)$ such that mean error squared $E^2[e(n)]$, where $E[\ ]$ is the expectation operator, is minimized. The minimization procedure applies a correction equal to a step-size multiplied by the negative of a vector of partial derivatives to the parameters of $C(z)$.

$$w(n+1) = w(n) - \mu\nabla(n) \tag{5}$$

where $w(n)$ is the coefficient vector at n$^{th}$ iteration and $\mu$ the step-size. $\nabla(n)$, the gradient at n$^{th}$ iteration, is

$$\begin{aligned}\nabla(n) &= \frac{\partial}{\partial w(n)} E^2[e(n)] \\ &= 2\cdot E[e(n)]\frac{\partial}{\partial w(n)} E[y_1(n)y_2(n)] \\ &= 2\cdot E[e(n)]E[y_1(n)\frac{\partial}{\partial w}\{y_2(n)\}]\end{aligned} \tag{6}$$

as $P(z)$ is not adapted by the adaptation procedure. From figure(3), $y_2$(n) is given by the difference equation

$$\begin{aligned}y_2(n) = {}& b_{0c}x_2(n-p) + b_{1c}x_2(n-p-1) \\ & -a_{0c}y_2(n-1)\end{aligned} \tag{7}$$

where $p$ is the delay in the Hilbert Transform-pair, $y_2(n)$ and $x_2(n)$ are the output and input sequences respectively and $b_{0c}$, $b_{1c}$ and $a_{0c}$ are the coefficients to be updated by the adaptation procedure. Taking the partial derivatives of $y_2(n)$ with respect to the coefficients $b_{0c}(n)$, $b_{1c}(n)$and $a_{0c}(n)$, adaptation of $k$ (in the present case, 2) numerator coefficients is done as follows:

$$\begin{aligned}b_{ic}(n) = {}& b_{ic}(n-1) - 2\mu y_1(n-1) \\ & \cdot E[e(n-1)]X_{ic}(n-1)\,,\ 0\le i<k\end{aligned} \tag{8}$$

where $X_{ic}(n-1) = x_2\left(n-i-p-1\right)$

$$-\sum_{l=1}^{m} a_{(l-1)c}(n-1)X_{ic}(n-1-l).$$

where $m$ is the number of denominator coefficients (in the present case, 1).

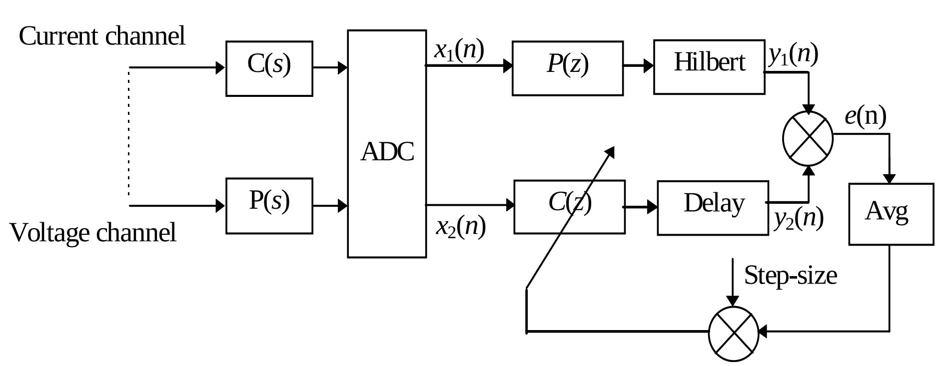



Figure 3. Phase estimation procedure.

The adaptation of $m$ denominator coefficients is as follows:

$$a_{jc}(n) = a_{jc}(n-1) - 2\mu y_1(n-1) \cdot E[e(n-1)]Y_{jc}(n-1)\ ,\ 0 \le j < m \qquad (9)$$

where

$$Y_{jc}(n-1) = -[\sum_{l=1}^{m} a_{(l-1)c}(n-1)Y_{jc}(n-1-l)] - y_2(n-j-2)$$

and at $n=0$, $X_{ic}(n\text{-}1\text{-}l)$ and $Y_{jc}(n\text{-}1\text{-}l)$ are initialized to zero for all $i$, $j$ and $l$.

The rate of convergence is governed by the step-size $\mu$ and the power at the adaptive filter's inputs. If the step-size is "normalized" according to the inverse ratio of the product of the CT and VT output variances, then the convergence rate becomes insensitive to these variances. Thus, the algorithm uses a variable step-size $\mu$ as follows:

$$\mu(n) = \frac{a}{\varphi(n)} \qquad (10)$$

where $a$ is a pre-determined constant and $\varphi$(n) is the product of estimates of the variances of the inputs $x_1(n)$ and $x_2(n)$.

$$\varphi(n) = \varphi_1(n)\varphi_2(n) \qquad (11)$$

where $\varphi_1(n) = (1-b)\varphi_1(n-1) + bx_1^2(n)$

$$\varphi_2(n) = (1-b)\varphi_2(n-1) + bx_2^2(n)$$

The excitation signal is obtained from equipment used to generate inputs for the fast transient burst test as per section 5.5.4 of [1]. A typical equipment generates a gated 1 MHz bandwidth noise signal at a 400 Hz pulse repetition rate. To couple this signal to a CT, a coupling network is usually required (in practice, this network introduces negligible phase shift in the frequencies of interest).

## 3. Experimental results

In an representative example, we choose

$$C(s) = \frac{600s}{1500.0231s + 210} \qquad (12)$$

$$P(s) = \frac{398 \times 10^3 s}{0.916s^2 + 3.98 \times 10^6 s + 286 \times 10^3} \qquad (13)$$

Figure 4 shows a Lissajous figure using the outputs of the filters, before and after adaptation, with a clipped sinewave input. while Figure 5 depicts the rate at which the error reduces.

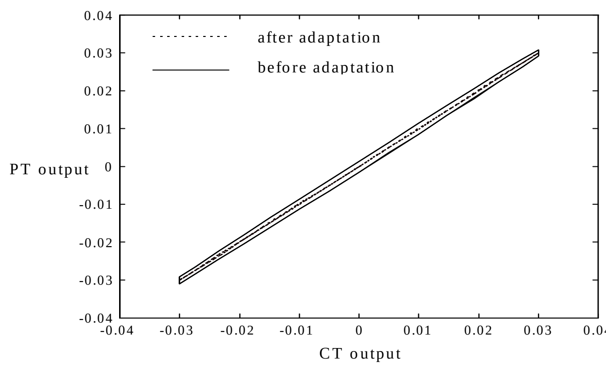



Figure 4. Lissajous figure for PT and CT outputs

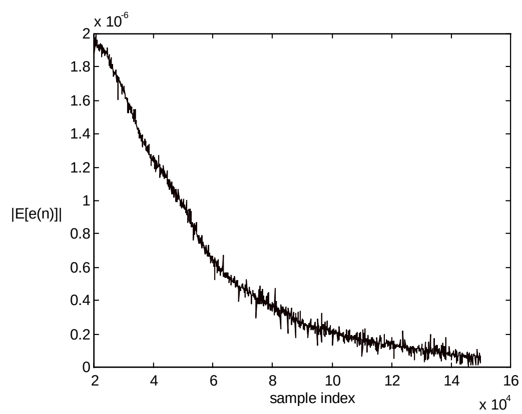



Figure 5. Error as a function of sample index

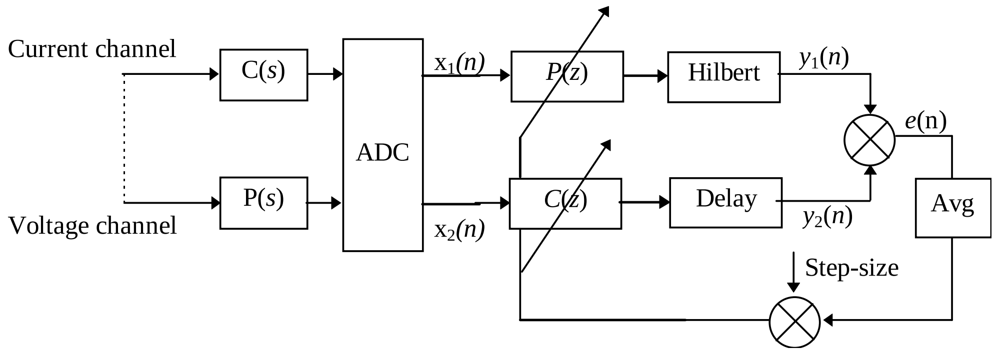



Figure 6. Phase estimation procedure when both C(*s*) and P(*s*) are unknown

## 4. Adaptive IIR structure for phase equalization when both C(*s*) and P(*s*) are unknown

As shown in figure 6 above, when both P(*s*) and C(*s*) are unknown, the adaptive scheme may also be used to estimate *P*(z) (replace all *c* subscripts by *p*'s and interchange the subscripts of *x* and *y* in (8) and (9)). However, *P*(*z*) and *C*(*z*) do not necessarily converge to $H_c(z)$ and $H_p(z)$ as phase is equalized even when they both multiply an arbitrary transfer function[2]. In practise, it was observed that when *C*(*z*) and *P*(*z*) are initialized to identity (i.e., assume ideal PT/ CT), the adapted filter coefficients are in fact scaled versions of $H_c(z)$ and $H_p(z)$, thus achieving phase equalization. Amplitude compensation (at the harmonic frequencies) is carried out in a further stage of calibration.

## 5. Conclusions

An adaptive IIR filter structure for all-pass phase equalization, its performance and its application to electronic energy metering are presented. A calibration procedure that equalizes CT and PT phase mismatches using standard test equipment is also provided.

[2] If *P*(*s*) and *C*(*s*) have no common factors and minimal orders selected for the adaptive filters *P*(*z*) and *C*(*z*), then *B*(*z*) can only be an arbitrary gain.

## References


[1] IEC-687, *Specification. for alternating current static watt-hour meters for active energy (classes 0.5 and 0.2)*, 1992.

[2] IEC-1036, *Specification for alternating current static watt-hour meters for active energy (classes 1 and 2)*, 1990.

[3] ANSI C12.16, *American National Standard for Electricity Metering- Solid-State Electricity Meters*, 1991.

[4] *Specifications for A.C static Electrical Energy Meters*, Central Board of Irrigation and Power, New Delhi, Technical Report No.88, December 1992.

[5] British Standard: BS5406:Part 2, 1988, *Disturbances in supply systems caused by household appliances and similar electrical equipment* (equivalent CENELEC standard is EN60555-2:1987)

[6] L.R Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*, pp. 216-224, Prentice Hall of India Private Limited, New Delhi, 1988.

[7] J.J. Shynk, "Adaptive IIR filtering", *IEEE ASSP Magazine*, No. 6, pp. 4-21, April 1989.